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Control modo deslizante-Δ modulación de un convertidor "buck"

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Resumen

Un enfoque propuesto recientemente que enmarca control de modos deslizantes de sistemas lineales dentro de un contexto de control clásico entrada/salida (Sira-Ramírez [8]), es usado aqui para la estabilización entrada-salida modo deslizante de un convertidor de potencia DC-DC tipo ''buck''. El enfoque evade mediciones de estado y evita el uso explícito de observadores asintóticos de estado en la síntesis de la superficie deslizante. El esquema propuesto es también robusto con respecto a entradas de perturbación ''unmatched''. Se resalta la conexión entre moduladores-Δ clásicos y los de modos deslizantes como una herramienta para la realización de esquemas de control deslizantes en sistemas comandados por un interruptor de posición donde un diseño de control de realimentación promedio deseable requiere ser implementado.

Palabras Claves: Control PI generalizado, Control de Modo Deslizante, Moduladores Delta.

Sliding mode-Δ modulation control of a "buck" converter

Abstract

A recently proposed approach that sets sliding mode control of linear systems within a classical input-output control framework (Sira-Ramírez [8]), is here used for the input-output sliding mode stabilization of "buck" DC-to-DC Power Converter. The approach evades state measurements and circumvents the explicit use of asymptotic state observers in the sliding surface synthesis. The proposed scheme is also robust with respect to unmatched perturbation inputs. A connection between sliding modes and classical Δ-modulators is also brought to attention as a tool for the realization of sliding mode control schemes in systems commanded by a switch position where a desirable average feedback controller design needs to be implemented.

Key words: Generalized PI control, Sliding Mode Control, Delta-modulators.

1. INTRODUCTION

The many advantages of sliding mode control are well reported, founded, and illustrated, in the existing literature. The sliding mode control technique is, fundamentally, a state space-based discontinuous feedback control technique. The lack of complete knowledge of the state vector components forces the designer to use asymptotic state observers, of the Luenberger, or of the sliding mode type, or perhaps to resort to direct output feedback control schemes. Unfortunately, the first approach is not robust with respect to unforseen exogenous perturbation inputs, even if they happen to be of the "classical type" (by this we mean: steps, ramps, parabolas, etc). The second approach is quite limited in nature and it is not applicable in a host of non-minimum phase systems. Generally speaking, state space based sliding mode techniques fail in the *unmatched perturbation input* case. For general background on sliding mode control, we refer the reader to the seminal books by Utkin [11], [12], the recent books by Utkin, Guldner and Shi [13] and that by Edwards and Spurgeon [1]. Recent developments, advances and applications of the sliding mode control area are found in the book by Perruquetti and Barbot [4].

In this article, we propose a new approach for the synthesis of sliding mode feedback control schemes for linear, time invariant, controllable and observable Single Input Single Output (SISO) systems, for which an average controller design is assumed to be available. We show that the use of classical

?-modulators allows for the switched synthesis of a feedback controller which has been synthesized within an *average* context (i.e. assuming that the control input continuously takes values on a closed subset of the real line, usually restricted to be the closed interval $[0,1]$). A complete account of ?-modulators, extensively used in analog signal encoding, which never benefited from the theoretical basis of sliding mode control, is found in the classical book by Steele [10].

We show that a ?-modulator can be used to translate such a continuous design into a discontinuous one with the property that the "equivalent output" signal of the modulator, in an ideal sliding mode sense, precisely matches the modulator's input signal.

When we combine ?-modulation with *integral reconstructors* of the system state vector and Generalized Proportional Integral (GPI) control, the result is that the required sliding motion is *dynamically* synthesized using only the input and the output of the system while retaining the essential robustness features of the average devised GPI controller.

Integral reconstructors were introduced in the work of Fliess [2], within the realm of continuous dynamic input output GPI feedback strategies. A complete theoretical account of integral reconstructions and GPI control has been presented in a recent article by Fliess *et al* [3]. In a rather different context than the one here presented, in [6], use is made of integral reconstructors in the synthesis of traditional inductor current based sliding surface for the boost converter (i.e. a minimum phase sliding surface coordinate function). A more complete set of Power Converters is also treated, from that viewpoint, [9]. Integral reconstructors require only inputs, outputs and iterated integrals of such signals for their synthesis, while neglecting the influence of the constant, but unknown, initial conditions and the effect of classical additive perturbation inputs. These effects are later counteracted by additional appropriate iterated integral error feedback in the controller expression. Hence, our approach is, fundamentally, an input-output approach which emphasizes the synthesis of an adequate continuous (average) feedback law, rather than the synthesis of a sliding surface. The use of GPI controllers in the average feedback controller design naturally leads to *dynamic* input-output feedback schemes for the synthesis of the sliding motion. As an outcome, the scheme here presented requires no "matching conditions" whatsoever.

Section 2 presents a review of the simplest analog ?-modulator and its connection with sliding mode control schemes when the actual system input signal takes values in a discrete set of the "ON-OFF" form, i.e. in the discrete set $\{0,1\}$. Section 3 deals with some generalities on how to synthesize a sliding mode controller on the basis of a given continuous feedback controller design. Section 4 concentrates on a direct application of ?-modulators in the implementation of a continuous Generalized PI controller for a "buck" converter model. Section 5 deals with the conclusions of the article.

2. ?-MODULATORS

Consider the basic block diagram of Figure 1 reminiscent of a ?-modulator block but with a binary valued forward nonlinearity, taking values in the discrete set $\{0,1\}$. For ease of reference we address such a block simply as a *?-Modulator*. The following theorem summarizes the relation of the depicted ?-modulator with sliding mode control while establishing the basic features of its input output performance.

Theorem 2.1: Consider the ?-modulator of Figure 1. Given a bounded C^1 signal $x(t)$, with bounded first order time derivative, $\dot{x}$, then the locally decoded feedback signal $x(t)$ satisfies the convergence property: $x(t) \rightarrow x(t)$, in a finite amount of time T_h,

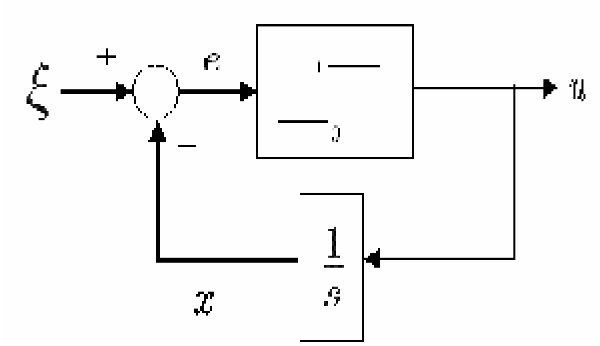

Figure 1. Switch position based ?-modulator.

provided the following *encoding condition* is satisfied for all t,

$$0 < \dot{x}(t) < 1 \tag{2.1}$$

Moreover, from any arbitrary initial value of the tracking, or encoding, error $e(t_0) = x(t_0) - x(t_0)$, a sliding motion exists on the perfect encoding condition

$e = 0$ for all $t > T_h$, where the quantity T_h is bounded by $T_h \leq T$, with T satisfying,

$$T \leq t_0 + \frac{2\,|\,e(t_0)\,|}{r(t)}$$

$$r(t) = (1 - \sup\{\dot{x}(t)\})(1 + \mathrm{sign}(e(t_0)))$$

$$+(\inf \dot{x}(t))(1 - \mathrm{sign}(e(t_0))) \qquad (2.2)$$

Proof. From the figure, the variables in the ? -modulator satisfy the following relations:

$$\dot{x} = u$$

$$u = \frac{1}{2}\left[1 + \mathrm{sign}(x - x)\right]$$

$$e = x - x \qquad (2.3)$$

Clearly, $\dot{e} = \dot{x}(t) - \frac{1}{2}\left[1 + \mathrm{sign}\,(e)\right]$ and since $\dot{x}(t)$ is assumed to be bounded within the interval $[0,1]$, we have:

$$e\dot{e} = e\dot{x}(t) - \frac{1}{2}e - \frac{1}{2}|e|$$

$$= -\frac{1}{2}|e|\left[1 - (2\dot{x}(t) - 1)\,\mathrm{sign}\,(e)\right]$$

$$\leq -\frac{1}{2}|\,e\,| < 0 \qquad (2.4)$$

A sliding regime exists on $e = 0$ for all time t after the hitting time T_h (see [11]). Under ideal sliding, or encoding, conditions, $e = 0, \dot{e} = 0$, we have that $x = x(t)$ and the *equivalent* (average) value of the coded output signal u is given by $u_{eq} = \dot{x}(t)$ for all $t \geq T_h$. The estimation of the hitting time T_h is based on the integrable error equation obtained for the most unfavorable case of the input $\dot{x}$ in the error equation for each possible sign of the initial condition. Thus, if $e(t_0) > 0$ then we consider as the error dynamics: $\dot{e} = \sup\{\dot{x}\} - 1$, which yields at the hitting time T_h,

$$e(T_h) = e(t_0) - (1 - \sup\{\dot{x}(t)\})(T_h - t_0) = 0 \Rightarrow$$

$$T_h = t_0 + \frac{e(t_0)}{1 - \sup\{\dot{x}(t)\}}$$

While if $e(t_0) < 0$, then we consider as the error dynamics $\dot{e} = \inf\{\dot{x}\}$. This yields, at the hitting time,

$$e(T_h) = e(t_0) + \inf\{\dot{x}\}(T_h - t_0) = 0 \Rightarrow$$

$$T_h = t_0 - \frac{e(t_0)}{\inf\{\dot{x}(t)\}} = t_0 + \frac{|e(t_0)|}{\inf\{\dot{x}(t)\}}$$

The result (2.2) follows by combining these two worst case estimates for the hitting time into a single formula which takes into account the sign of the initial condition $e(t_0)$.

The ? -modulator output u ideally yields the time differentiation of the modulator input signal $x(t)$ in an *equivalent control* sense [11], i.e. in an *average sense*. The role of the above described ? -modulator in sliding mode control schemes, avoiding full state measurements, and using average based controllers will be clear from the examples presented below.

3. USE OF A DELTA MODULATOR IN SLIDING MODE CONTROL IMPLEMENTATION OF AN AVERAGE CONTROLLER DESIGN

Suppose we have a smooth nonlinear system of the form $\dot{x} = f(x) + ug(x)$ with u being a (continuous) control input signal that, for some physical limitations, requires to be bounded by the closed interval $[0,1]$. Suppose, furthermore, that we have been able to specify a state feedback controller of the form $u = -k(x)$, or a dynamic output feedback controller of the form $u = -k(y,z), \dot{z} = j(y,z)$, with desirable closed loop performance features. Assume, furthermore, that for any initial state of the system (and of the dynamic controller, if such is the case), the values of the feedback signal function, $u(t)$, are uniformly strictly bounded by the closed interval $[0,1]$.

If an additional implementation requirement entitles now that the control input u of the system is no longer allowed to continuously take values within the interval $[0,1]$, but that it may only take values in the *discrete set*, $\{0,1\}$, the natural question is: how can we now implement the derived continuous controller, so that we can recover, possibly in an *average sense*, the desirable features of the derived static, or dynamic, feedback controller design in view of the last imposed actuator restriction?

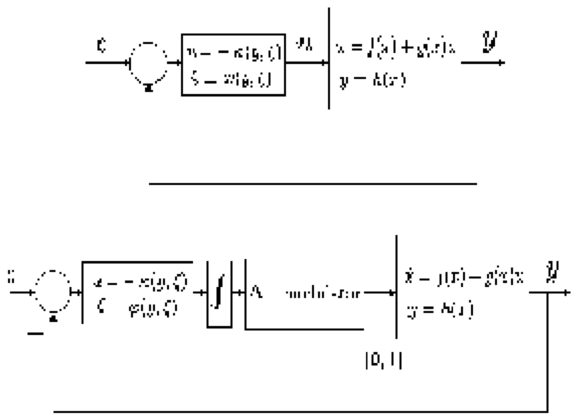

Figure 2. Sliding mode implementation of a designed continuous output feedback controller through a ?-modulator.

The answer is clearly given by the average differentiating features of the input signals in the previously considered ?-modulator. Recall, incidentally, that the output signal of such a modulator is restricted to take values, precisely, in the discrete set $\{0,1\}$. Thus, if the *time integral* of the output of the designed continuous controller, call it $u_{av}(t)$, is fed into the proposed Delta modulator, the output signal of the modulator reproduces, on the average, the required control signal u_{av}. Figure 2 shows the switch based implementation of an output feedback controller, through a ?-modulator, which reproduces, in an average sense, the features of a designed continuous controller. Note that the presence of an integral in the feedback loop of the ?-modulator and the requirement of the integral of the designed average control signal at the input of the modulator produces the following equivalence entitling a single integration placed before the sign function (see Figure 3). We have the following alternative equations for the delta modulator when the input signal x is to be reproduced at the output of the modulator in an average, equivalent control, sense.

$$e = x - u$$
$$u = \frac{1}{2}[1+\text{sign } z]$$
$$\dot{z} = e \tag{3.1}$$

We have the following proposition.

Theorem 3.1: Consider the modified ?-modulator of Figure 3. Given a bounded signal $x(t)$, then the integral tracking error z converges to zero in a finite amount of time T_h, provided the following *encoding condition* is satisfied for all t,

$$0 < x(t) < 1 \tag{3.2}$$

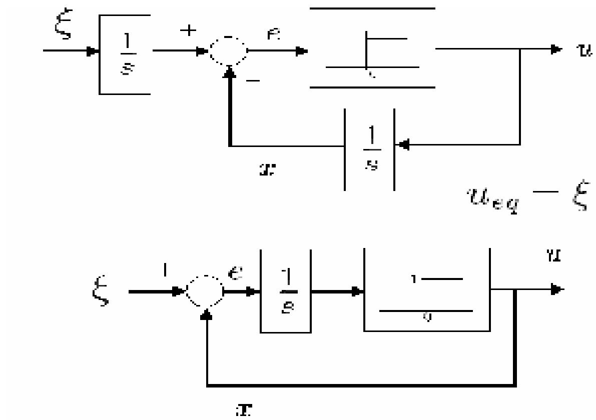

Figure 3. Equivalence of ?-modulator schemes for average control law implementation.

Moreover, a sliding motion exists on the perfect encoding condition $z=0$ for all $t > T_h$, where $T_h \leq T$, with T satisfying,

$$T \leq t_0 + \frac{2\,|z(t_0)|}{r(t)}$$
$$r(t) = (1-\sup\{x(t)\})(1+\text{sign}(z(t_0))) + (\inf x(t))(1-\text{sign}(z(t_0))) \tag{3.3}$$

Proof. From the modified modulator equations (3.1), we have

$$\frac{dz}{dt} = x(t) - \frac{1}{2}\left[1+\text{sign }(z)\right] \tag{3.4}$$

and therefore

$$z\dot{z} = -\frac{1}{2}|z|\left[1+(1-2x(t))\text{sign }(z)\right] \tag{3.5}$$

Clearly, for $x(t)$ being a bounded signal in the open interval $(0,1)$, it follows that $z\dot{z} < 0$, and a sliding regime exists on $z=0$. Under ideal sliding motions, the conditions: $z=0$, $dz/dt=0$ are valid. This implies that, on the average, $x(t)=u(t)$, i.e. the output of the modified modulator reproduces, in an equivalent control sense, the input to the modulator. The finite time reachability of the sliding surface $z=0$ is established in a similar manner as in the previous theorem.

In view of the previous result, we have the following general result concerning the control of nonlinear systems through sliding modes synthesized on the basis of an average feedback controller and a Δ-modulator. We only deal with the dynamic output feedback controller case.

Theorem 3.2: Consider the following smooth nonlinear single input, *n*-dimensional system: $\dot{x} = f(x) + ug(x)$, with the smooth scalar output map, $y = h(x)$. Assume the dynamic smooth output feedback controller $u = -k(y,z), \dot{z} = j(y,z)$, with $z \in R^p$, locally (globally) asymptotically stabilizes the system to a desired constant equilibrium state, denoted by X. Assume, furthermore, that the control signal, u, is uniformly strictly bounded by the closed interval [0,1] of the real line. Then the closed loop system:

$$\begin{aligned}
\dot{x} &= f(x) + ug(x) \\
y &= h(x) \\
u_{av}(y,z) &= -k(y,z) \\
\dot{z} &= j(y,z) \\
u &= \frac{1}{2}[1 + \text{sign } z] \\
\dot{z} &= u_{av}(y,z) - u
\end{aligned}$$

exhibits an *ideal sliding dynamics* which is locally (globally) asymptotically stable to the same constant state equilibrium point, X, of the system.

Proof. The proof of this theorem is immediate upon realizing that under the hypothesis on the average control input, u_{av}, the previous theorem establishes that a sliding regime exists on the manifold $z = 0$. Under the *invariance conditions*, $z = 0, \dot{z} = 0$, which characterize ideal sliding motions (See Sira-Ramírez [5]), the corresponding *equivalent control*, u_{eq}, associated, with the system satisfies: $u_{eq}(t) = u_{av}(t)$. The *ideal sliding dynamics* is then represented by

$$\begin{aligned}
\dot{x} &= f(x) + u_{av}g(x) \\
y &= h(x) \\
u_{av}(y,z) &= -k(y,z) \\
\dot{z} &= j(y,z)
\end{aligned}$$

which is assumed to be locally (globally) asymptotically stable towards the desired equilibrium point.

4. CONTROL OF A "BUCK" CONVERTER CIRCUIT

A. The "buck" converter model, its average model and a PD controller.

Consider the "buck" converter circuit shown in Figure 4. The system is described by the set of equations

$$\begin{aligned}
L\dot{i} &= -v + uE \\
C\dot{v} &= i - \frac{v}{R}
\end{aligned} \tag{4.1}$$

where i represents the inductor current and v is the output capacitor voltage. The control input **u**, representing the switch position function, is a discrete-valued signal taking values in the set {0,1}. The system parameters are constituted by: L, which is the inductance of the input circuit; C**,** the capacitance of the output filter and R, the output load resistance. The external voltage source has the constant value E. We assume that the circuit is in continuous conduction mode, i.e. the average value of the inductor current never drops to zero, due to load variations.

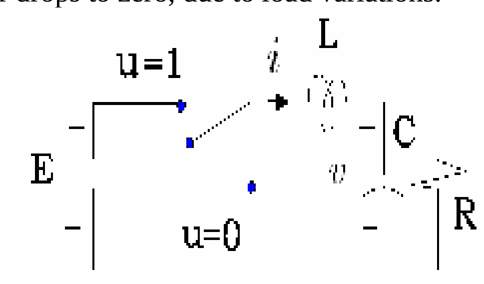

Figure 4. Switch regulated DC-to-DC "buck" power converter.

We introduce the following state normalization and time scale transformation:

$$x_1 = \frac{i}{E}\sqrt{\frac{L}{C}}, \quad x_2 = \frac{v}{E}, \quad t = \frac{t}{\sqrt{LC}} \tag{4.2}$$

The normalized model is thus given by:

$$\begin{aligned}
\dot{x}_1 &= -x_2 + u \\
\dot{x}_2 &= x_1 - \frac{x_2}{Q}
\end{aligned} \tag{4.3}$$

where now, with an abuse of notation, the " $\cdot$ " represents derivation with respect to the normalized time, t . The variable x_1 is the normalized inductor current, x_2 is the normalized output voltage and u, still represents the switch position function. The constant system parameters are all comprised now in the circuit "quality" parameter, denoted by Q and given by the strictly positive quantity, $R\sqrt{C/L}$. We assume that only the (normalized) output capacitor voltage, $y = x_2$, is available for measurement.

In order to obtain a suitable average controller, assume for a moment that the normalized "buck" converter equations actually represent a continuous system (i.e. an average system) where u may take values in the closed interval [0,1]. Take the normalized "average output" capacitor voltage, x_2, as the system output, i.e. $y = x_2$. Elimination of the normalized "average inductor current" variable x_1 leads to the following input-output differential relation,

$$\ddot{y} + \frac{1}{Q}\dot{y} + y = u \qquad (4.4)$$

Clearly, the system, aside from being a stable system, it also has no zero dynamics associated with $\dot{y}$. Suppose we want to devise a controller that asymptotically regulates the output voltage to the desired "average" value $\bar{y}$. Corresponding to this desired normalized constant equilibrium value, we have from (4.4) $\bar{u} = \bar{y}$.

Consider then, the following PD stabilizing feedback controller with nominal input compensation,

$$u = \bar{u} + \left(\frac{1}{Q} - k_2\right)\dot{y} + (1-k_1)(y-\bar{y}) \qquad (4.5)$$

The tracking error, $e = y - \bar{y}$, is clearly seen to satisfy the following closed loop dynamics

$$\ddot{e} + k_2\dot{e} + k_1 e = 0 \qquad (4.6)$$

By appropriate choice of the design parameters k_2, k_1, the origin of the error space coordinate, e, can be made into a locally asymptotically exponentially stable equilibrium point determined by the non-saturation condition.

$$0 < \bar{u} + \left(\frac{1}{Q} - k_2\right)\dot{y} + (1-k_1)(y-\bar{y}) < 1 \qquad (4.7)$$

B. An average Generalized PI (GPI) control for the "buck" converter.

The PD controller (4.5) requires the time derivative of the output signal y. A GPI controller can then be proposed which substitutes the unmeasured signal $\dot{y}$ by its *integral reconstructor*, denoted by $(\dot{y})_e$. Such a reconstructor, is obtained by direct integration of the input output relation (4.4), while neglecting the unknown initial condition $\dot{y}_0$. We have,

$$(\dot{y})_e = -\frac{1}{Q}y - \int_0^t (y(s))ds \qquad (4.8)$$

The use of $(\dot{y})_e$ in the PD controller, instead of $\dot{y}$, produces a constant error due to the unaccounted relation, $\dot{y} = (\dot{y})_e + \dot{y}_0$. An integral control corrective action is then added to the proposed controller (4.5), in order to counteract the conscientious neglect of the unknown initial condition. We propose then the following average dynamic GPI feedback controller,

$$u = \bar{u} + \left(\frac{1}{Q} - k_2\right)(\dot{y})_e + (1-k_1)(y-\bar{y})$$
$$+ k_0\int_0^t (y(s) - \bar{y})ds$$
$$(\dot{y})_e = -\frac{1}{Q}y - \int_0^t (y(s) - u)ds \qquad (4.9)$$

The closed loop system stabilization error dynamics satisfies the following linear integro-differential equation excited by an unknown constant.

$$\ddot{e} + k_1\dot{e} + k_0\int_0^t e(s)ds = \left[\frac{1}{Q} - k_2\right]\dot{y}_0 \qquad (4.10)$$

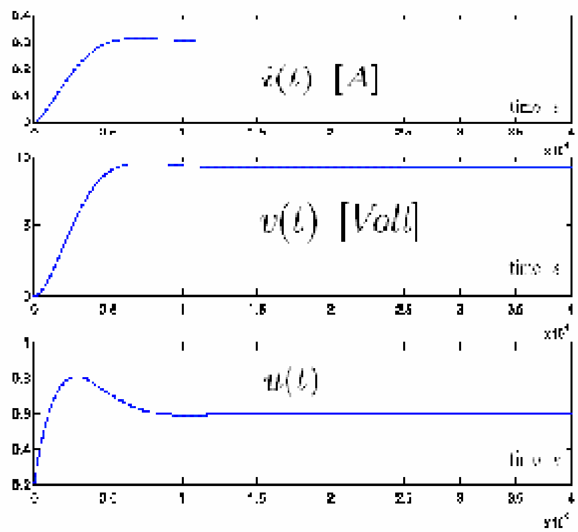

Figure 5. Average system response to continuous (average) GPI feedback controller design.

It is not difficult to verify that such a closed loop dynamics has the origin as an asymptotically exponentially stable equilibrium point for a suitable set of design parameters k_2, k_1, k_0. Indeed, define a new state x by means of

$$x = \int_0^t e(s)ds - \frac{1}{k_0}\left(\frac{1}{Q} - k_2\right)\dot{y}_0 \qquad (4.11)$$

The closed loop system is then equivalent to the following composite linear system,

$$\ddot{e} + k_1\dot{e} + k_0 x = 0$$

$$\dot{x} = e(s), \quad x(0) = -\frac{1}{k_0}\left(\frac{1}{Q} - k_2\right)\dot{y}_0$$

Clearly, the characteristic polynomial of the closed loop system is given by

$$p(s) = s^3 + k_2 s^2 + k_1 s + k_0 \qquad (4.12)$$

Figure 5 shows some computer simulations of the closed loop response of the non normalized converter system variables *v*, *i*, to the actions of the proposed average GPI controller. The controller design parameters k_2, k_1, k_0, were chosen so that the average control input u_{av} never saturates to values outside the interval [0,1]. For this, we set: $k_2 = 3p, k_1 = 3p^2, k_0 = p^3$ with $p = 0.4$, in the normalized system simulations. The converter parameters were taken to be.

$$L = 10^{-3}[H], \qquad C = 10^{-6}[Farad],$$
$$R = 30[Ohm], \qquad E = 15[V]$$

For these parameters, the time normalization factor is found to be $\sqrt{LC} = 3.1622x10^{-5}$

C. GPI-sliding mode control of the "buck" converter implemented through a D-modulator.

Based on the presented justifications, we propose to implement the GPI controller (4.9) on the "buck" power converter by means of a Δ-modulator, as previously discussed.

$$u = \frac{1}{2}(1 + sign(z))$$

$$\dot{z} = e = u_{av} - u$$

$$u_{av} = \bar{u} + \left(\frac{1}{Q} - k_2\right)(\hat{\dot{y}})_e + (1 - k_1)(y - \bar{y})$$
$$+ k_0\int_0^t (y(s) - \bar{y})ds$$

$$(\hat{\dot{y}})_e = -\frac{1}{Q}y - \int_0^t (y(s) - u)ds$$

Under ideal sliding conditions on the sliding surface, $z = 0$, the corresponding dynamics is precisely represented by the condition $u = u_{av}$. The stability analysis carried out for the closed loop behavior of the average system under a GPI controller thus becomes valid. The sliding mode controller results in the origin of the tracking error, $y - \bar{y}$, to be an exponentially asymptotically stable equilibrium point for all motions that do not saturate the control input $u_{av}(t)$ beyond the interval $[0,1]$.

Figure 6 shows computer simulations depicting the closed loop response of the system to the actions of the GPI controller implemented through a Δ-modulator. The controller design parameters and the system parameters were chosen to be exactly the same as those used in the previous simulation of the average GPI feedback controlled responses. In order to test the robustness of the Δ-modulator implementation of the proposed average GPI controller, we tested the system with an unmatched sudden constant perturbation, denoted by $h1(t-t)$, appearing at time $t = 0.004$ $[s]$ of value $h = 0.6667$ [A]. i.e. we used the model:

$$L\dot{i} = -v + uE$$

$$C\dot{v} = i - \frac{v}{R} + n1(t - t) \qquad (4.13)$$

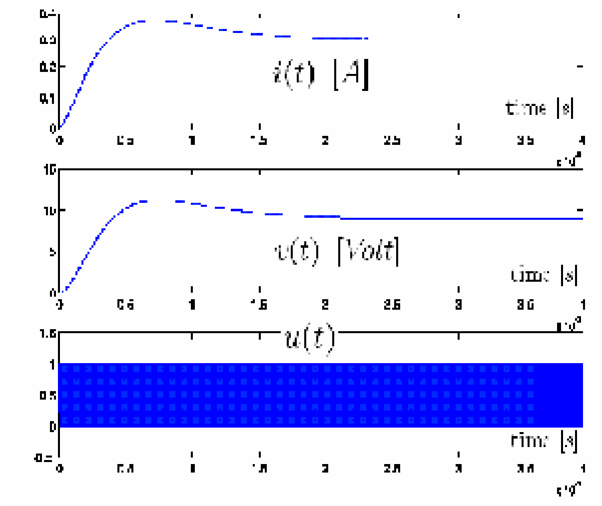

Figure 6. Closed loop response of a "buck" to converter to a Δ-modulator implementation a of GPI stabilizing controller.

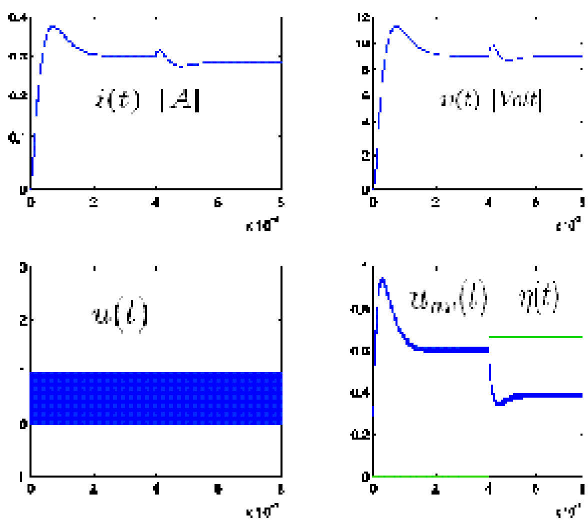

Figure 7. Robustness of a Sliding mode-Δ modulator implementation of a GPI stabilizing controller acting on a "buck" converter undergoing an unmatched step perturbation input.

Figure 7 shows computer simulations depicting the closed loop response and the recovery features of the implemented GPI-Δ-modulation controller. Ideal sliding motions in traditional sliding mode control do not inherit the robustness features nor are they robust with respect to "unmatched" perturbations.

5. CONCLUSIONS

Average feedback controller designs usually represent the desirable *equivalent control* in sliding mode control implementations. The exact synthesis of the equivalent control is not physically possible in systems commanded by switches, and sign non-linearities, such as in traditional, and double bridge, DC-to-DC power converters. Knowledge of the feedback law defining the equivalent control leads to consider a linear partial differential equation, for the sliding surface, stating that the closed loop vector field should be orthogonal to the sliding surface gradient. However, it is still not obvious how to synthesize a sliding surface, that corresponds to a given equivalent control, due to the indeterminacy, and arbitrariness, of the boundary conditions in the defining linear partial differential equation that needs to be solved.

In this article, we have demonstrated that the use of classical Δ-modulators can solve the sliding mode implementation problem of average feedback controllers in a rather efficient manner. The proposed approach retains, in an average sense, the desirable features of the designed average feedback controller. When the proposed controllers are synthesized using only inputs and outputs, as in GPI control, the explicit asymptotic estimation of the state becomes unnecessary and, moreover, the *matching conditions*, intimately related to the state space representation of the system, are no longer needed.

We have used the Δ-modulator implementation of a sliding mode controller for a given average GPI continuous controller in a "buck" DC-to-DC power converter. Other non-linear switched controlled systems may immediately benefit form the sliding mode feedback controller design framework based on Δ-modulators and nonlinear output feedback controllers arising from current nonlinear systems theory (for instance, geometric, differential algebraic, flatness, passivity, energy methods, H_∞, etc.). For DC-to-DC power converters, in particular, average passivity based output feedback controllers, such as those developed in [7] may be readily implemented via sliding mode controllers in a direct fashion.

6. ACKNOWLEDGMENT

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