Una propuesta de control por modos deslizantes para procesos con tiempo muerto variable

Oscar Camacho, Rubén Rojas, Luís González
Escuela de Ingeniería Eléctrica, Universidad de los Andes, Mérida-Venezuela
Email: ocamacho@ula.ve

Resumen

Este artículo presenta la manera de mejorar la respuesta transitoria en controladores por modos deslizantes para procesos con tiempo muerto variable. El controlador por modos deslizantes está basado sobre un modelo de proceso del tipo primer orden más tiempo muerto. El desarrollo consiste en agregar una conmutación al término integral de la superficie deslizante cuando los errores o parámetros del modelo se modifican debido a un cambio en la entrada del proceso. El controlador es sintonizado con un conjunto de ecuaciones en función de parámetros del modelo del proceso. Los resultados son mostrados sobre un proceso no lineal con tiempo muerto variable.

Palabras clave: Control por modo deslizante, tiempo muerto variable, control robusto.

Sliding mode control proposal for variable dead time processes

Abstract

This paper aims to show a way to improve the transient response of sliding mode controllers for variable dead time processes. The sliding mode controller is based on a first-order-plus-dead time model of the process. The improvement consists in switching the integral term of the sliding surface when modeling errors or the parameters change with the process inputs. The controller is a fixed structure with a set of tuning equations as a function of the characteristic parameters of the model. Simulations on a nonlinear process with variable dead time judge the controller performance.

Keywords: Sliding mode control, variable delay time, robust control

1. INTRODUCTION

In process control, time delay is the time that it takes for the output variable to respond after a change in the control signal has occurred. Possible sources of time delays are: A.- The process may involve the transportation of materials or fluids over long distances. B.- The measuring device may be subject to long delays to provide a measurement. C.- The final control element may need some time to develop the actuating signal. The presence of time delays causes the following difficulties in process control: A disturbance entering the process will not be detected until after a significant period of time. The control action will be inadequate since its effects on a current error will affect the process variable only after a long delay. The gain and phase margins are strongly affected by delay time. Long time delays may originate instability in the system [1].

Process deadtime is recognized as one of the major problems in process control. Different solutions have been applied beginning with the Smith Predictor, its modifications and other Model Based Controllers (MBC) [2]. In most of the cases mentioned above, the controllers can provide a good control for deadtime systems, when the process model matches the process reasonably well.

Many important processes are characterized by response with dominant transport delay. This kind of processes suffer a great impact on control performance when present changes are in response delay. Many controllers uses a model identified through process testing.

Those models are linear and time variant, if the assumptions are true the control generated from a process model identified during normal conditions will perform well. On the other hand, if the process
dynamics changes then such changes affect the control, degrading its performance. Therefore, the control performance degrades if the process response changes from that assumed in the process model. One approach to solve the problem is to use multiple models and different tuning for each process condition, given a switching condition among models and tuning values, but it is complicated from maintenance, of multiple models and control conditions, point of view. Therefore, MBC are extremely sensitive on deadtime variations, producing cycling responses, evidently longer deadtime systems requires more robust controllers.

Designing controllers that, in spite of the uncertainty present in the model, can deliver high performance, achieving asymptotic tracking and disturbance rejection, and also guarantee system stability is a robust control problem. In previous paper [3, 4], it has been shown that Sliding Mode Control (SMC), is a kind of robust control that can solve the previous problems.

This paper aims to show a way to improve the transient response of Sliding Mode Controllers for variable deadtime process. The Sliding Mode Controller is based on a first-order-plus-deadtime model of the process. The approach consists in switching the integral term of the sliding surface (ISSMCr), when modeling errors or the parameters change with the process inputs. The controller is of fixed structure with a set of tuning equations as a function of the characteristic parameters of the model. Simulations on a nonlinear chemical process with variable deadtime judge the controller performance

This article is organized as follows. Section 2 presents some basic concepts of the SMC method. Section 3 shows the SMCr equations, obtained from an FOPDT model, so as the tuning. Section 4 describes the ISSMCr modification. In Section 5 simulations of both SMCrs are presented. An ISE index is used to compare both controllers, the results are also presented. In Section 6 conclusions are summarized.

2. BASIC CONCEPTS ABOUT SLIDING MODE CONTROL

Sliding Mode Control is a technique derived from Variable Structure Control (VSC) which was originally studied by Utkin[5]. The controller designed using the SMC method is particularly appealing due to its ability to deal with nonlinear systems and time-varying systems [6],[7],[8]. The robustness to the uncertainties becomes an important aspect in designing any control system.

The idea behind SMC is to define a surface along which the process can slide to its desired final value. The structure of the controller is intentionally altered as its state crosses the surface in accordance with a prescribed control law. Thus, the first step in SMC is to define the sliding surface $S(t)$ that is linear and stable. The $S(t)$ selected in this work (1), presented by [8], is an integral-differential equation acting on the tracking-error expression:

$$S(t) = \left(\frac{d}{dt} + \lambda\right)^n \int_0^t e(t)dt$$  \hspace{1cm} (1)

Where $e(t)$ is tracking error, that is, the difference between the reference value or set point, $R(t)$, and the output measurement, $X(t)$, or $e(t) = R(t) - X(t)$. $\lambda$ is a tuning parameter, which helps to define $S(t)$, this term is selected by the designer, and determines the performance of the system on the sliding surface, $n$ is the system’s order.

The objective of control is to ensure that the controlled variable is equal to its reference value at all times, meaning that $e(t)$ and its derivatives must be zero. Once the reference value is reached, (1) indicates that $S(t)$ reaches a constant value. To maintain $S(t)$ at this constant value, meaning that $e(t)$ is zero at all times; it is desired to make:

$$\frac{dS(t)}{dt} = 0$$  \hspace{1cm} (2)

Once the sliding surface has been selected, attention must be turned to design of the control law that drives the controlled variable to its reference value and satisfies (2). The SMC control law, $U(t)$, consists of two additive parts (3); a continuous part, $U_c(t)$, and a discontinuous part, $U_d(t)$ [9]. That is:

$$U(t) = U_c(t) + U_d(t)$$  \hspace{1cm} (3)

The continuous part is given by (4).

$$U_c(t) = f \left[ X(t), R(t) \right]$$  \hspace{1cm} (4)
Where \( f[X(t), R(t)] \) is a function of the controlled variable, and the reference value.

The discontinuous part (5), UD(t), incorporates a nonlinear element that includes the switching element of the control law. This part of the controller is discontinuous across the sliding surface.

\[
U_d(t) = K_D \frac{S(t)}{S(t) + \delta} \tag{5}
\]

Where KD is the tuning parameter responsible for the reaching mode. d is a tuning parameter used to reduce the chattering problem. Chattering is a nondecreasing oscillatory component of finite amplitude and frequency. It is undesirable in practice, because it involves high control activity and also can excite high frequency dynamics ignored in the modeling or identifying process [7],[8],[9].

In summary, the control law usually results in a fast motion to bring the state onto the sliding surface, and a slower motion to proceed until a desired state is reached.

3. SMCR FROM AN FOPDT MODEL OF THE PROCESS

This section presents the general equation of the SMCr, for self-regulating processes. This equation was derived using a first-order-plus-dead-time (FOPDT) process model, it can be found in [4]. The controller obtained significantly simplifies the application of sliding mode control theory to industrial processes.

Then, from [4], the complete SMCr in (6).

\[
U(t) = \left( \frac{t_0 \tau}{K} \right) \left[ \frac{X(t)}{t_0 \tau} + \lambda_a e(t) \right] + K_D \frac{S(t)}{S(t) + \delta} \tag{6}
\]

with

\[
S(t) = \text{sign}(K) \left[ \frac{dX(t)}{dt} + \lambda_a e(t) + \lambda_c \int_0^t e(t) dt \right] \tag{7}
\]

The term \( \text{sign}(K) \) in (7) takes into consideration the sign of the process gain and its use results in the correct controller action. It never switches; thus (7) is basically a PID algorithm.

To complete the controller the following tuning equations are offered [3]:

- For the continuous part of the controller and the sliding surface (8) and (9).

\[
\lambda_c = \frac{t_a + \tau}{t_a \tau} \quad \text{[time]1} \tag{8}
\]

\[
\lambda_c \leq \frac{\lambda_a^2}{4} \quad \text{[time]2} \tag{9}
\]

- For the discontinuous part of the controller:

\[
K_D = \frac{0.51}{K} \left[ \frac{\tau}{t_a} \right] \quad \text{[fraction CO]} \tag{11}
\]

\[
\delta = 0.68 + 0.12(K/K_D \lambda_a) \quad \text{[fraction CO]} \tag{12}
\]

Equation (11) and (12) are used when the signals from the transmitter and controller are in fractions 0 to 1. Sometimes, the control systems work in percentages that are, the signals are in \( \% \), 0 to 100 of range. In these cases the values of \( K_D \) and \( \delta \) are multiplied by 100. Figure 1 shows the original SMCr scheme.

4. THE INTEGRAL SWITCHING SLIDING MODE CONTROLLER (ISSMCR)

The strategy that improves the controller clock performance is based on two modules. They are: A Detection Module (DM), designed to issue a pulse when the controlled variable is on the reference...
value. Which means \( e(t) = 0 \) and \( de/dt \neq 0 \). The second module is called Compensation Module (CM), it is responsible of accumulating the value of \( U_D(t) \) in the moment when the detection module emits a pulse, and then send the accumulated value to \( U(t) \) through \( D(t) \), as is shown in Figure 2.

Other important function, of DM, is to indicate when is necessary to clean the accumulated value in the integral part of the sliding surface. The procedure can be explained as follows: when CM add the accumulated value to \( U(t) \), the integral part of the sliding surface is zero, in such a way that the controller output does not present any hard change. Therefore it permits using \( U_D(t) \) nearby to the origin, where its slope is the highest as is shown in Figure 3.

5. SIMULATION RESULTS

This section shows by computer simulations the performance of the ISSMCr and the SMCr. The process is a mixing tank with variable deadtime depending of the inlet flow [4], the principal characteristic of this example is that the controllability relationship to/t>1, which increases when a disturbance in the hot water occurs.

5.1. Mixing Tank

Consider the mixing tank shown in Figure 4. The tank receives two streams, a hot stream, \( W_1(t) \), and a cold stream, \( W_2(t) \). The outlet temperature is measured at a point 125 ft downstream from the tank. The following assumptions are accepted

- The liquid volume in the tank is considered constant
- The tank contents are well mixed
- The tank and the pipe are well insulated.
- The temperature transmitter is calibrated for a range of 100°F to 200°F. Table 1 shows the steady-state conditions and other operating information.

The following equations constitute the process model:

- Energy balance around mixing tank:
  \[
  W_1(t)Cp_1(t)T_1(t) - W_2(t)Cp_2(t)T_2(t) - \left( W_1(t) + W_2(t) \right)Cp_3(t)T_3(t) = \rho CV_3 \frac{dT_3(t)}{dt}
  \]
  (13)

- Pipe delay between the tank and the sensor location:
  \[
  T_4(t) = T_3(t)(t - t_d(t))
  \]
  (14)
Camacho, Rojas y González

- Transportation lag or delay time:
  \[ t_o = \frac{L A \rho}{W_1(t) + W_2(t)} \]  

- Temperature Transmitter:
  \[ \frac{dT(t)}{dt} = \frac{1}{\tau_T}((T_1(t) - 100) - TO(t)) \]  

- Valve position:
  \[ \frac{dV_p(t)}{dt} = \frac{1}{\tau_{V_p}}(\frac{1}{100}(m(t)) - V_p(t)) \]  

- Valve equation:
  \[ W_2(t) = \frac{500}{60} C_v V_p(t) \sqrt{G_f \Delta P_v} \]  

- Sliding Mode Controller (SMCr):
  \[ U(t) = U_c(t) + U_s(t) \]

where
- \( W_1(t) \) = mass flow of hot stream, lb/min
- \( W_2(t) \) = mass flow of cold stream, lb/min
- \( C_p \) = liquid heat capacity at constant pressure, Btu/lb-°F
- \( C_v \) = liquid heat capacity at constant volume, Btu/lb-°F
- \( T_1(t) \) = hot flow temperature, °F
- \( T_2(t) \) = cold flow temperature, °F
- \( T_3(t) \) = liquid temperature in the mixing tank, °F
- \( T_4(t) \) = equal to \( T_3(t) \) delayed by \( t_o \), °F
- \( t_o \) = deadtime or transportation lag, min
- \( \rho \) = density of the mixing tank contents, lbm/ft³
- \( V \) = liquid volume, ft³
- \( \rho \) = density of the mixing tank contents, lbm/ft³
- \( V \) = liquid volume, ft³
- \( TO(t) \) = transmitter output signal on a scale from 0 to 1
- \( V_p(t) \) = valve position, from 0 (valve closed) to 1 (valve open)
- \( m(t) \) = fraction of controller output, from 0 to 1
- \( C_{vl} \) = valve flow coefficient, gpm/psi¹²
- \( \tau_T \) = time constant of the temperature sensor, min
- \( \tau_{V_p} \) = time constant of the actuator, min
- \( A \) = pipe cross section, ft²
- \( L \) = pipe length, ft

Using the reaction curve method [1] the characteristic parameters of the process are obtained, there are: \( K = -0.78 \) fraction TO/fraction CO τ = 2.32 min., and \( t_o = 2.97 \) min.

Table 1. Design parameters and steady-state values.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_1 )</td>
<td>250.00 lb/min</td>
<td>( V )</td>
<td>15 ft³</td>
</tr>
<tr>
<td>( W_2 )</td>
<td>191.17 lb/min</td>
<td>( TO )</td>
<td>0.5</td>
</tr>
<tr>
<td>( C_p_1 )</td>
<td>0.8 Btu/lb-°F</td>
<td>( V_p )</td>
<td>0.478</td>
</tr>
<tr>
<td>( C_p_2 )</td>
<td>1.0 Btu/lb-°F</td>
<td>( C_{vl} )</td>
<td>12 gpm/psi¹²</td>
</tr>
<tr>
<td>( C_p_3, C_v_3 )</td>
<td>0.9 Btu/lb-°F</td>
<td>( \Delta P_v )</td>
<td>16 psi</td>
</tr>
<tr>
<td>Set point</td>
<td>150 °F</td>
<td>( \tau_T )</td>
<td>0.5 min</td>
</tr>
<tr>
<td>( T_1 )</td>
<td>250 °F</td>
<td>( \tau_{v_p} )</td>
<td>0.4 min</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>50 °F</td>
<td>( A )</td>
<td>0.2006 ft²</td>
</tr>
<tr>
<td>( T_3 )</td>
<td>150 °F</td>
<td>( L )</td>
<td>125 ft</td>
</tr>
<tr>
<td>( \rho )</td>
<td>62.4 lb/ft³</td>
<td>( \overline{m} )</td>
<td>0.478 CO</td>
</tr>
</tbody>
</table>

From these values, and using the tuning equations, both controllers are tuned as follows (20):

\[
\lambda_1 = 0.767 \text{ min}^{-1}; \quad K_D = 0.54 \text{ fraction TO} \]
\[
\lambda_0 = 0.147 \text{ min}^{-2}; \quad \delta = 0.719 \text{ fraction TO} \text{ / min} \]

Please note that the controller equations, Eqs. 24a and 24b, were developed using deviation variables. The following changes the “deviation variables” in the controller to “actual variables”

\[
U(t) = m(t) - \overline{m} \]  

\[
X(t) = TO(t) - \overline{TO} \]

and

\[
e(t) = R(t) - TO(t) \]

where \( m(t) \) is the controller output, in fraction CO, \( TO(t) \) is the transmitter output, in fraction, and \( R(t) \) is the reference value, or set point, fraction TO. The overbars indicate steady-state values.

Since the process gain is negative, \( \text{sign}(K) \) is negative, the controller equation to be used is:

\[
m(t) = \overline{m} - \frac{t_o}{K} \left( \frac{1}{t_o} (TO(t) - \overline{TO}) + \lambda_m R(t) \right) + \frac{K_D S}{S} \]

(24a)
with

\[ S(t) = \frac{dTO(t)}{dt} - \lambda_1 e(t) - \lambda_0 \int_0^t e(t) dt \]  

(24b)

5.2. Results without noise

The controllers' responses are shown in Figures 5a and 5b, without modeling errors, and in the Table 2, a comparative analysis, based on the ISE performing index can be seen.

where DE(%) (25):

\[ \Delta E(\%) = \frac{[ISSMC - SMC]}{(ISSMC + SMC)/2} \times 100\% \]  

(25)

\[ \Delta E(\%)^* = \frac{[ISSMCr - SMCr]}{(ISSMCr + SMCr)/2} \times 100\% \]  

(25)

Table 2. Comparative analysis.

<table>
<thead>
<tr>
<th></th>
<th>ISE (1 Step)</th>
<th>ISE (2 Step)</th>
<th>ISE (3 Step)</th>
<th>ISE (4 Step)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISSMCr</td>
<td>0.4823</td>
<td>0.1500</td>
<td>0.2135</td>
<td>0.4152</td>
</tr>
<tr>
<td>SMCr</td>
<td>0.4823</td>
<td>0.2069</td>
<td>0.3204</td>
<td>0.5318</td>
</tr>
<tr>
<td>$\Delta E(%)^*$</td>
<td>0</td>
<td>31.88</td>
<td>40.04</td>
<td>24.62</td>
</tr>
</tbody>
</table>

Figures 6 and 7 and Table 3,4, depict process and controller output. The figures show that for errors of 20% in $K$, and $r_o$, the new approach presents better performance than the old one.

Table 3. Comparative Analysis for +20% in $K$

<table>
<thead>
<tr>
<th></th>
<th>ISE (1 Step)</th>
<th>ISE (2 Step)</th>
<th>ISE (3 Step)</th>
<th>ISE (4 Step)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISSMCr</td>
<td>0.5835</td>
<td>0.1731</td>
<td>0.2329</td>
<td>0.3837</td>
</tr>
<tr>
<td>SMCr</td>
<td>0.5835</td>
<td>0.2670</td>
<td>0.4330</td>
<td>0.7589</td>
</tr>
<tr>
<td>$\Delta E(%)^*$</td>
<td>0</td>
<td>42.67</td>
<td>60.09</td>
<td>65.67</td>
</tr>
</tbody>
</table>

Table 4. Comparative Analysis for +20% in $r_o$

<table>
<thead>
<tr>
<th></th>
<th>ISE (1 Step)</th>
<th>ISE (2 Step)</th>
<th>ISE (3 Step)</th>
<th>ISE (4 Step)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISSMCr</td>
<td>0.6001</td>
<td>0.1755</td>
<td>0.2296</td>
<td>0.3578</td>
</tr>
<tr>
<td>SMCr</td>
<td>0.6001</td>
<td>0.2680</td>
<td>0.4251</td>
<td>0.7213</td>
</tr>
<tr>
<td>$\Delta E(%)^*$</td>
<td>0</td>
<td>41.71</td>
<td>59.72</td>
<td>67.37</td>
</tr>
</tbody>
</table>
5.3. Results with noise

In spite of noise present in the process, the results are similar to the previous case, as can be seen in figures (8a),(8b),(9) and (10).

6. CONCLUSIONS

This paper has shown an approach to improve the performance of a Sliding Mode Controller when a process presents variable parameters. This approach can be implemented using DCS. The controller modification adds the integral switching term, but the tunings are similar to the traditional version.
REFERENCES


